Minimum Weight Design of Cylindrical Shell with Multiple Stiffener Sizes

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The minimum weight design for the simply supported orthogonally stiffened cylindrical shell with smeared-out and discrete stiffeners subjected to axial compression is studied by a method of multipliers. The method for constraint function minimization proposed by Fletcher and Powell and the quasi-Newton method are used and compared. The difficulty in the formidably expensive computations of eigenvalues in the optimization process is circumvented by an approximate buckling formulation using extremely simple displacement functions. Expensive exact eigensolution is, however, performed for the optimized design variables to check the buckling load obtained by the approximate method. Seven design variables and fourteen inequality constraints are used for a design with a single stiffener size. Eleven design variables and twenty-one inequality constraints are used for a design with two stiffener sizes. The two designs are compared and discussed. From the calculations presented the design with two stiffener sizes can be lighter than the one with one stiffener size.

Introduction

THE problems of minimum weight design of stiffened cylindrical shells under buckling constraints have long been of interest to aerospace structural designers. Studies of such problems are extensive.[1-17]

Before 1967, the studies[1-3] were based on the assumption of simultaneous failure modes and the optimization was achieved by parametric studies. The structural syntheses for the subject problems were performed, among others, by Kicher,[4] Schmit et al.[5] and Pappas and Amba-Rao.[8] Kicher[4] treated the problem by using the constrained gradient method. Schmit et al.[5] applied to a Fiacco-McCormick type penalty function formulation to transform the basic inequality constrained minimization problem into a sequency of unconstrained minimization problem. Pappas and Amba-Rao[8] used a direct search algorithm with an interior-exterior penalty function formulation.

Thornton[12] performed the synthesis of stiffened conical shells using the exterior penalty function method with least-square approximation. Kunoo and Yang[17] carried out the minimum weight design of cylindrical shells with orthogonal stiffeners subjected to uniform axial compressive or bending load by the method of steepest descent. Hofmeister and Felton[18,19] showed that the waffle plate with multiple rib sizes compared more favorably to sandwich plates than those with a single stiffener size.

In all of the references[1-17] concerned with the weight optimization of the stiffened shells, it appears that all of the buckling predictions were made by using the formulations based on the assumption that the stiffeners may be considered as continuously distributed (smeared-out) over the shell reference surface. The main reason for not using discrete design and discrete formulation is that they result in large sets of eigenvalue equations that are extremely expensive to solve repeatedly during optimization.

A survey of the development in the buckling analysis of cylindrical shells with smeared-out and discrete stiffeners has been conducted by the present writers.[20] Exact buckling formulations for the cylindrical shells with inner and outer orthogonal discrete stiffeners are provided.

By assuming equally spaced and identical discrete stringers and rings, the buckling formulations could be uncoupled into simpler ones based on their associated mode shapes. Then, following a matrix partitioning and reduction procedure, an efficient use of matrix sparsities, and the use of an efficient eigenvalue solution procedure, the huge sets of eigenvalue equations could be further reduced and became manageable.[20] It was demonstrated in a typical example of cylinder with discrete stiffeners that, through the use of such procedures, approximately 400 sec of central processing time was needed for computing one lowest eigenvalue by a CDC 6500 computer. This example problem was governed by 750 eigenvalue equations in the original coupled formulation. The computing time of 400 sec is, however, still formidably long in the minimum weight design where repeated computations of the lowest eigenvalues are required.

To circumvent this difficulty, it is suggested here to assume extremely simple displacement functions in the buckling formulation. With such simplicity, the computing time required for repeated eigenvalue computations in the optimization process becomes small. To insure that the optimized design is feasible, the exact buckling load based on the exact discrete formulation[20] is finally computed using the optimized design variables to check the buckling load obtained by the approximate method.

A method of multipliers recently developed by Schuldt[21] for mathematical programming problems is extended to the present structural problems for efficient minimum weight design. In order to illustrate the present simplified method for buckling analysis and the method of multipliers for optimization, two design examples of simply supported axially compressed cylindrical shells are presented, one with the smeared-out stiffeners and one with both the smeared-out and the discrete stiffeners.

Weight, Constraints, and Design Variables

The weight W of an orthogonally stiffened shell with multiple stiffener sizes is expressed as: weight = weight of the skin + weight of the smeared-out stringers + weight of the smeared-out rings − weight of the material at the intersections of the smeared-out stringers and rings + weight of the discrete stringers + weight of the discrete rings − weight of the material at the intersections of the discrete stringers and

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rings; or

$$W=2\pi rLh\rho+2\pi rLh_{e1}\rho+\pi[2r-\mathrm{sgn}(d_2)(d_2+h)]h_{e2}L\rho$$
$$-\min(|d_1|,|d_2|)\{1+\mathrm{sgn}(d_1\cdot d_2)\}/2$$
$$\cdot h_{e1}h_{e2}/|d_1\cdot d_2|+L\rho\sum_{K=1}^{n_s}A_{s_K}+\pi[2r-\mathrm{sgn}(d_r)(d_r+h)]\rho$$
$$\cdot\sum_{K=1}^{n_r}A_{r_K}-\min(|d_s|,|d_r|)\{1+\mathrm{sgn}(d_s\cdot d_r)\}/2t_st_rn_sn_r\rho \quad (1)$$

with

$$h_{e1}=d_1t_1/a_1,\quad h_{e2}=d_2t_2(1/a_2-1/L),\quad \mathrm{sgn}(x)=x/|x| \quad (2)$$

where r, h, L are the mean radius, thickness, and length of the cylindrical shell, respectively; ρ is the material density; d and t are the depth and width of the stiffeners, respectively; the subscripts 1, 2, s, and r are associated with the smeared-out stringer, the smeared-out ring, the discrete stringer, and the discrete ring, respectively; a is the spacing between the stiffeners; A and n are the cross-sectional area and the total number of the stiffeners.

For the cylindrical shell with orthogonal smeared-out stiffeners the inequality constraints are defined as follows:

1) Buckling of the stiffened cylinder

$$\psi_1=1-P_x/P_{x0}\geq 0 \quad (3)$$

where P_{x0} is the general buckling load.

2) Buckling of the stiffened cylinder between the circumferential rings with two circular edges considered as simply supported

$$\psi_2=1-P_{xp}/P_{x0}\geq 0 \quad (4)$$

3) Buckling of the shell skin bounded by longitudinal and circumferential stiffeners with four edges considered as simply supported

$$\psi_3=\frac{\pi^2h^2E}{3(1-\nu^2)a_1{}^2}\div\sigma_x-1\geq 0 \quad (5)$$

4) Skin failure by yielding

$$\psi_4=1-(\sigma_x{}^2-\sigma_x\sigma_\theta+\sigma_\theta{}^2)/\sigma_y{}^2\geq 0 \quad (6)$$

where σ_y is the yielding stress of the skin.

5) Longitudinal stringer failure by yielding

$$\psi_5=1-\sigma_{x1}/\sigma_y\geq 0 \quad (7)$$

6) Circumferential ring failure by yielding

$$\psi_6=1-|\sigma_{\theta 2}|/\sigma_y\geq 0 \quad (8)$$

7) Buckling of the longitudinal stringer between the circumferential rings with one end free and the other end simply supported[22]

$$\psi_7=\frac{\pi^2E}{12(1-\nu^2)}\left(\frac{t_1}{d_1}\right)^2\left[\left(\frac{d_1}{a_2}\right)^2+0.425\right]\div\sigma_{x1}-1\geq 0 \quad (9)$$

8) Design variable constraints

$$u_{l_i}\leq u_i\leq u_{u_i}\qquad i=1,2,\ldots p \quad (10)$$

or

$$\psi=(u_i-u_{l_i})(u_{u_i}-u_i)\geq 0 \quad (11)$$

where the design variables consist of a_1, a_2, t_1, t_2, d_1, d_2, and h.

It is assumed in Eq. (9) that both stringers and rings are on the same side of the skin and that the rings are deeper than the stringers. If these conditions are not satisfied, the spacing a_2 in Eq. (9) should be replaced by the spacing between two discrete rings $L/(1+n_r)$ on the same side of the skin. In the absence of discrete rings, n_r is zero.

According to the foregoing definitions, the design of the cylindrical shell with orthogonal smeared-out stiffeners includes seven design variables and fourteen inequality constraints. For the cylindrical shell with orthogonal smeared-out and discrete stiffeners, the inequality constraints described for the smeared-out case still hold, but the computation of the general buckling load P_{x0} used in Eqs. (3) and (4) should incorporate the effect of discrete stiffeners as well. Besides, three more constraints are included:

1) Longitudinal discrete stringer failure by yielding

$$\psi=1-\sigma_{xs}/\sigma_{ys}\geq 0 \quad (12)$$

2) Circumferential discrete ring failure by yielding

$$\psi=1-|\sigma_{\theta r}|/\sigma_{yr}\geq 0 \quad (13)$$

3) Buckling of the longitudinal discrete stringer between two circumferential rings with one end free and the other end simply supported[22]

$$\psi=\frac{\pi^2E_s}{12(1-\nu_s^2)}\left(\frac{t_s}{d_s}\right)^2\left[\left\{\frac{d_s(1+n_r)}{L}\right\}^2+0.425\right]\div\sigma_{xs}-1\geq 0 \quad (14)$$

It is assumed in Eq. (14) that both stringers and rings are on the same side of the skin and that the rings are deeper than the stringers. If these conditions are violated, n_r should be set equal to zero.

The design variables chosen for this case are $a_1,a_2,t_1,t_2,d_1,d_2,h,t_s,d_s,t_r$, and d_r. The design of the cylindrical shell with both smeared-out and discrete stiffeners thus includes 11 design variables and 21 inequality constraints.

A Simplified Buckling Formulation

It has been shown in example 2 (Table 3) of Ref. 20 that for the buckling analysis of a simply supported cylindrical shell with one ring and four stringers under uniform axial compression, the exact formulation required a minimum of 750 eigenvalue equations (with $k_m'=k_m=25$ and $k_n'=k_n=5$). With the use of a matrix uncoupling technique based on the associated mode shapes, a matrix reduction technique by partitioning and substitution, a special compact storage scheme to make use of the sparsities of the submatrices, and the Ritz iteration method combined with the Chebyshev eigenvalue solution procedure, the lowest eigenvalue for the 750 equations was found eventually by a CDC 6500 computer with only approximately 400 sec of central processing time. Although it is already a tremendous reduction in computing time, 400 sec is still formidably long if the buckling load has to be computed repeatedly for optimization.

To circumvent this difficulty, the computing time for buckling load must be further reduced drastically, and it appears that one has no alternative but to seek for an extremely simple and short method for only a rough estimate of buckling load. In that case the exact buckling computation still has, of course, to be performed for the optimized design variables to check the validity of the buckling load obtained by the approximate method.

In the case of cylinder with smeared-out stiffeners, it has been shown[23] that an exact solution can be achieved by using one term for each u, v, and w functions. Thus, for extreme simplicity, one antisymmetric circumferential mode and one longitudinal mode are assumed in the displacement functions for the cylinder with discrete stiffeners.

$$u = U_{mn} \cos(m\pi x/L) \sin n\theta \tag{15a}$$

$$v = V_{mn} \sin(m\pi x/L) \cos n\theta \tag{15b}$$

$$w = W_{mn} \sin(m\pi x/L) \sin n\theta \tag{15c}$$

The displacement functions are substituted into the potential energy expressions [Eq. (1) in Ref. 20] and the same procedure as shown in Ref. 20 is followed for deriving the buckling equations. The resulting equations are similar to the ones for the cylinder with smeared-out stiffeners and its explicit formulations can be obtained by setting $m = m_i = m_j$ and $n = n_i = n_j$ in Eq. (25) of Ref. 20. The critical buckling load is obtained by searching for the smallest eigenvalue in a set of three equations by varying the values of integers m and n.

In the case of cylinders with smeared-out stiffeners a smooth surface for the minimum eigenvalue is obtained so that the efficient sectioning approach proposed and demonstrated in Ref. 17 may be used for the search for the minimum. In the case of cylinders with discrete stiffeners, there are many local minima in the eigenvalue surface. The absolute minimum has to be searched for by varying m and n continuously in a certain range. In spite of this disadvantage, this simple approximate discrete calculation is superior to the smeared-out calculation in the buckling prediction of a cylinder with discrete stiffeners.

This approximate discrete method was first verified by performing a buckling computation of a cylinder with 100 stringers and 16 rings. The material and geometry parameters of the cylinder are defined as: $E = 1.06 \times 10^7$ psi, $\nu = 1/3$, $a_1 = a_2 = 3$ in., $d_1 = d_2 = 0.2$ in., $t_1 = t_2 = 0.125$ in., $h = 0.05$ in., $r = 48$ in., and $L = 50$ in. The exact solution obtained by the exact discrete formulation was given in Ref. 20. The buckling load was 801.3 lb/in. and the buckling mode numbers were $m = 6$ and $n = 17$. The present approximate discrete computation gave a buckling load of 807.9 lb/in. with the mode numbers $m = 6$ and $n = 17$. It is noted that in order to obtain the converged solutions by discrete formulations in Ref. 20, 600 degrees-of-freedom were required, while in this approximate method only three degrees-of-freedom were used.

This approximate discrete method was then tested for a cylinder with only one ring and four stringers. The material and geometry parameters of the cylinder are defined as: $E = 10.6 \times 10^6$ psi, $\nu = 1/3$, $h = 0.03$ in., $r = 48$ in., $L = 50$ in., $d_s = d_r = 1.3196$ in., $t_s = t_r = 0.8248$ in. The converged buckling stress was obtained in Ref. 20 as 4068 psi with several different mode shapes. The present discrete approximate method gave a buckling stress of 5008 psi with buckling mode numbers $m = 24$ and $n = 2$.

It is important to note that when the seven sets of near minimum eigenvalue solutions for the foregoing examples were obtained in Ref. 20, the sizes of the originally coupled eigenvalue equations were 504, 630, 750, 936, 1008, 1134, and 1248. If the present approximate method is used, only three eigenvalue equations need to be solved. The loss in accuracy is compensated for by the drastic reduction in computing time. Without such drastic reduction the optimization computations will be too expensive to perform. The buckling load for the optimized design variables obtained by this approximate method is, however, checked by the expensive exact discrete computation. The difference in the predicted buckling loads between this approximate discrete method and the exact discrete method becomes smaller as the number of stiffeners is increased. As will be shown in a subsequent example of minimum weight design, the buckling load predicted by this approximate method for a cylinder with 5 discrete rings and 30 discrete stringers is very close to the exact discrete solution.

Theory of Optimization

The theory of optimization has long been an attractive research topic and the developments are extensive. Various theories of optimization have been applied to the minimum weight design of structures with various constraints. These applications have been discussed in the Introduction.

Recently, Schuldt[21] developed a method of multipliers for mathematical programming problems with equality and inequality constraints. Since this method has not been applied to the minimum weight design of structures, it is introduced here as an efficient method for the minimum weight design of the stiffened cylindrical shell with multiple stiffener sizes.

The conventional problem of optimum design is to minimize the weight of a structure expressed as a function of the design variables which satisfy a series of constraints on the design behavior and on the sizes of the variables (design variable equality constraints or design variable inequality constraints). That is, to minimize $W(\bar{u})$ subject to

$$\bar{\phi}(\bar{u}) = \bar{0} \tag{16}$$

$$\bar{\psi}(\bar{u}) \geq \bar{0} \tag{17}$$

According to Ref. 21 the problem may be transformed into the one of minimizing the augmented function

$$F(\bar{u},\bar{\lambda},\bar{\mu}) = W(\bar{u}) + \bar{\lambda}^T\bar{\phi}(\bar{u}) - \bar{\mu}^T\bar{\psi}(\bar{u}) \tag{18}$$

subject to Eqs. (16) and (17) which satisfy the following first-order condition at the solution point.

$$F_u(\bar{u},\bar{\lambda},\bar{\mu}) = W_u(\bar{u}) + \bar{\phi}_u(\bar{u})\bar{\lambda} - \bar{\psi}_u(\bar{u})\bar{\mu} = 0 \tag{19}$$

$$\mu_{(j)}\psi_{(j)}(\bar{u}) = 0 \qquad j = 1,\ldots,r \tag{20}$$

$$\mu_{(j)} \geqq 0 \qquad j = 1,\ldots,r \tag{21}$$

The augmented penalty function is written as

$$\Omega(\bar{u},\bar{\lambda},\bar{\mu},k) = Q(\bar{u},\bar{\lambda},k) + R(\bar{u},\bar{\mu},k) \tag{22}$$

where

$$\begin{aligned} Q &= W(\bar{u}) + \bar{\lambda}^T\bar{\phi}(\bar{u}) + K\bar{\phi}^T(\bar{u})\bar{\phi}(\bar{u}) \\ &= W(\bar{u}) + K[\bar{\phi}(\bar{u}) + (1/2/k)\bar{\lambda}]^T \\ &\times [\bar{\phi}(\bar{u}) + (1/2/k)\bar{\lambda}] - (1/4/k)\bar{\lambda}^T\bar{\lambda} \end{aligned} \tag{23}$$

$$R = k[\tilde{\psi}(\bar{u}) - (1/2/k)\bar{\mu}]^T[\tilde{\psi}(\bar{u}) - (1/2/k)\bar{\mu}] - (1/4/k)\bar{\mu}^T\bar{\mu} \tag{24}$$

$$\tilde{\psi}_j(\bar{u}) = \min[\psi_{(j)}(\bar{u}), (1/2/k)\mu_{(j)}] \quad j = 1,\ldots,r \tag{25}$$

The constant k is initially chosen from scaling consideration and held fixed throughout the algorithm.

Initial cycle is begun with the choice of $\bar{\lambda} = \bar{0}$ and $\bar{\mu} = \bar{0}$, i.e.,

$$\Omega(\bar{u},\bar{0},\bar{0},k) = W(\bar{u}) + k[\bar{\phi}^T(\bar{u})\bar{\phi}(\bar{u}) + \tilde{\psi}^T(\bar{u})\tilde{\psi}(\bar{u})] \tag{26}$$

where

$$\tilde{\psi}_j(x) = \min\{\psi_j(x), 0\} \qquad j = 1,\ldots,r \tag{27}$$

The following multiplier updating rules are applied:

$$\bar{\lambda}_2 = \bar{\lambda}_1 + 2k\bar{\phi}(x) \tag{28}$$

$$\bar{\mu}_2 = \bar{\mu}_1 - 2k\tilde{\psi}(x) \tag{29}$$

where subscripts 1 and 2 correspond to the current and the next cycles, respectively.

Differentiating Eq. (22) with respect to the design variables $\bar{u}$, the following necessary conditions for the minimum is obtained,

$$\Omega_u(\bar{u},\bar{\lambda},\bar{\mu},k) = W_u(\bar{u}) + \bar{\phi}_u(\bar{u})[\bar{\lambda} + 2k\bar{\phi}(\bar{u})]$$
$$- \bar{\psi}(\bar{u})[\bar{\mu} - 2k\bar{\psi}(\bar{u})] = 0 \tag{30}$$

Combining Eqs. (28-30), one may obtain the following expression where the updated multipliers satisfy the optimality condition given by Eq. (19)

$$F_u(\bar{u},\bar{\lambda}_2,\bar{\mu}_2) = W_u(\bar{u}) + \bar{\phi}_u(\bar{u})\bar{\lambda}_2 - \bar{\psi}_u(\bar{u})\bar{\mu}_2 = 0 \tag{31}$$

In addition to this condition, the equality constraints by Eq. (16) and the inequality constraints defined by Eq. (17) should be satisfied. In Ref. 21 Schuldt first showed that the augmented penalty function increases at the end of each minimization cycle when the multipliers are changed from $\bar{\lambda}_1$ and $\bar{\mu}_1$ to $\bar{\lambda}_2$ and $\bar{\mu}_2$, respectively. He then demonstrated through one example that succeeding cycles of minimizing the augmented penalty function may drive toward constrained satisfaction.

In order to minimize the augmented penalty function Ω defined by Eq. (22), the method for the unconstrained function minimization proposed by Fletcher and Powell[24] and a quasi-Newton method are used. The two methods are also compared. All gradient calculations are carried out by the simple finite-difference method in this study.

Design Examples and Discussion

The first design example chosen was a simply supported orthogonally stiffened cylindrical shell subjected to uniform axial compression. The cylinder had a radius of 48 in. and a length of 50 in. The properties of the aluminum were used for both the cylinder and the stiffeners: $E = 10.6 \times 10^6$ psi, $\nu = 1/3$, $\rho = 0.101$ lb/in.[3] The buckling load of $P_{x0} = 238.2$ kips or $N_{x0} = 789.8$ lb/in. was used as the buckling constraint. The seven initial design variables which were restrained between 0.01 and 10 in. are shown in Table 1. This problem is similar to the one given in Ref. 17 but with one extra design variable, a_2.

The buckling computation was based on the simple smeared-out formulation and the optimization process was performed by the method of multipliers with the Fletcher-Powell method as well as the quasi-Newton method. One of the purposes of choosing this example was to obtain data for comparison with a later example with additional discrete stiffeners. The other purpose of conducting this example was to evaluate the performance of the present optimization methods. The results for the minimum weight design are presented in Table 1. For this example, the Fletcher-Powell method is seen to be more suitable than the quasi-Newton method.

The second example was a simply supported aluminum cylindrical shell with 5 discrete rings, 30 discrete stringers, and smeared-out stiffeners. The cylinder had the same length, radius, and material properties as the first example. The buckling constraint was 789.82 lb/in. The 11 initial design variables which were restrained between 0.01 and 10 in. are shown in Table 2.

The buckling computation during the weight minimization was performed using the present approximate discrete technique. The results of the minimum weight design are given in Table 2. The approximate buckling load for the minimum weight design obtained by using the Fletcher-Powell method was also checked by using the exact buckling computation based on the discrete formulation.[20] The symmetrical longitudinal modes with $m = 1, 3, 5, \dots 59$, the antisymmetric circumferential modes, and the circumferentially related modes with $n = 7, 23, 37, 53$ governed the buckling equations. The exact buckling load obtained was 801.7 lb/in. which was very close to the 815.8 lb/in. obtained by the approximate discrete method. The corresponding mode shape is shown in Fig. 1. This optimized mode shape shows that a very smooth buckling surface resulted from the combined effect of the smeared-out stiffeners and the discrete stiffeners. Again, Table 2 reveals that the Fletcher-Powell method is more suitable in the present weight minimization than the quasi-Newton method.

The process of the weight minimization by using the Fletcher-Powell method is summarized in Table 3. The weight reduction was completed in the early stage of the optimization, but the violation of the constraints was sometimes greater than 10%. The rest of the optimization cycles were carried out to correct the violations. All of the final designs violated the constraints by less than 1%.

The optimized stiffened shell with single stiffener size has the failure modes of general buckling and buckling of the shell skin bounded by longitudinal and circumferential stiffeners, expressed by Eqs. (3) and (5), respectively. The optimized stiffened shell with multiple stiffener sizes is inside the feasible region and near the failure modes of general buckling, buckling of the shell skin, and buckling of the smeared-out longitudinal stringer, expressed by Eqs. (3, 5, and 9), respectively.

The present examples show that the method of multipliers with the Fletcher-Powell method is very effective in the weight minimization. The results in Table 3 show that under the same buckling constraint the cylinder with smeared-out and discrete stiffeners results in less weight than the cylinder with only smeared-out stiffeners.

Table 1 Weight minimization using the smeared-out formulation[a]

Iteration number	Number of function evaluations	Weight, lb	Buckling stress, N_x, lb/in.	Mode number m n	a_1, in.	a_2, in.
0	0	124.63	913.5	4 15	1.0	3.0
20	2334	45.23	789.5	6 7	0.5645	4.962
0	0	124.63	913.5	4 15	1.0	3.0
11	915	51.22	788.0	4 11	0.8125	4.454

d_1, in.	d_2, in.	t_1, in.	t_2, in.	h, in.	CDC 6500 CP time, sec	Method
0.2	0.2	0.125	0.125	0.05	0	Fletcher-
0.2737	1.7077	0.02287	0.00999	0.01559	2315	Powell
0.2	0.2	0.125	0.125	0.05	0	quasi-
0.3318	0.8337	0.02546	0.01181	0.02126	879	Newton

[a] $N_{x0} = 789.82$ lb/in., $\sigma_y = 50$ ksi, 0.01 in. $\leq u_i \leq 10$ in., $i = 1,2,\dots 7$.

Table 2 Weight minimization using smeared-out and discrete formulations[a]

Iteration number	NFE[b]	Weight, lb	N_x, lb/in.	Mode m n	a_1, in.	a_2, in.	d_1, in.	d_2, in.
0	0	141.21	1102.8	3 14	1.0000	3.0000	0.200	0.2000
15	1636	42.83	815.8	11 15	0.3321	2.7754	0.1715	1.0446
0	0	141.21	1102.8	3 14	1.0000	3.0000	0.2000	0.2000
11	934	78.01	792.1	7 15	1.0109	3.0003	0.1759	0.2619

t_1, in.	t_2, in.	h, in.	d_s, in.	t_s, in.	d_r, in.	t_r, in.	T,[c] sec	Method
0.1250	0.1250	0.0500	0.4085	0.2553	0.0913	0.0571	0	Fletcher-
0.0156	0.0110	0.0100	0.6938	0.0896	0.0145	0.0321	1711	Powell
0.1250	0.1250	0.0500	0.4085	0.2552	0.0913	0.0571	0	quasi
0.0361	0.1192	0.0230	0.4414	0.2741	0.0892	0.0509	841	Newton

[a] N_{x0} = 789.82 lb/in, σ_y = 50 ksi, 0.01 in. $\le u_i \le$ 10. in. with i = 1,2,...11, n_s = 30, n_r = 5. [b] Number of function evaluations. [c] CDC 6500 central processing time.

Table 3 Process of the weight minimization by the Fletcher-Powell method

Iteration number	Number of function evaluations	CDC 6500 CP time, sec	Weight, lb	Formulation
0	0	0	124.63	
5	538	594	54.50	
10	1098	1194	49.32	Smeared-out
14	1702	1709	46.38	
20	2334	2315	45.23	
0	0	0	141.21	
4	408	444	48.10	Smeared-out
9	912	968	45.62	and discrete
11	1144	1203	43.85	
15	1636	1711	42.83	

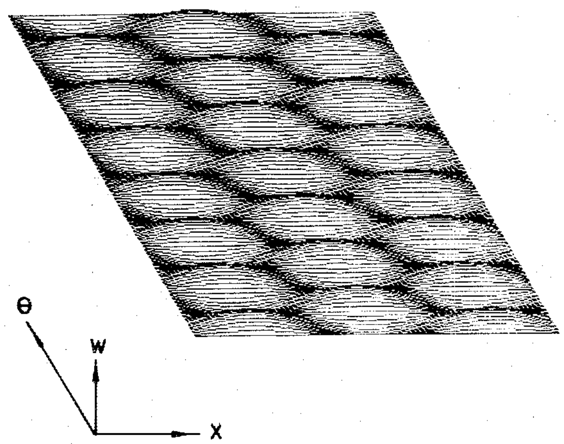

Fig. 1 Buckling mode shape for the optimized cylindrical shell with smeared-out and discrete stiffeners.

Concluding Remarks

The minimum weight design of orthogonally stiffened cylindrical shells with discrete stiffeners in addition to the smeared-out stiffeners has not been studied before, mainly due to the computational difficulty in obtaining the buckling load. With the development in Ref. 20, which formulated the discrete stiffeners exactly and simplified the computational method tremendously, such computational difficulty was reduced. The simplifications included uncoupling the buckling matrix based on the associated mode shapes, reducing the uncoupled submatrices by partitioning and substitution, taking advantage of the sparsities of the buckling matrix by a special compact storage scheme, and using an efficient eigenvalue solution procedure. The development is, however, still too expensive to be used repeatedly for weight minimization.

In this study, an approximate discrete formulation based on very simple displacement functions has been suggested for computing the buckling load for the discretely stiffened cylinders. This method requires the solution of a set of only three instread of several hundred eigenvalue equations. The difficulty in computing time for the weight minimization has thus been overcome. To insure that the optimized design using this approximate discrete method for predicting buckling loads is valid, the exact discrete buckling computation is still performed based on the final design variables.

To cope with this simple discrete method for buckling computation, the methods of multipliers with the Fletcher-Powell method and with quasi-Newton method have been chosen for the weight optimization. Two design examples have been performed.

The design examples have provided the following indications:

1) The present simple discrete method is very effective in weight minimization design. The second example shows that the buckling load predicted by this approximate method was very close to the exact solution.

2) The comparison shows that for unconstrained minimization the Fletcher-Powell method is more suitable than the quasi-Newton method. The former method is very effective in dealing with the present type of weight minimization problem.

3) For the minimum weight design of a cylindrical shell under axial compression, the cylinder with both the smeared-out and the discrete stiffeners can be lighter than the cylinder with only smeared-out stiffeners.

Finally, it should be noted that the foregoing indications are obtained from only two design examples and they should not be regarded as general conclusions.

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